$$f\ \frac{d^2y}{dx^2} + g\ \frac{dy}{dx} + h\ y = RHS$$

1 Constant Coefficients

Homogeneous $RHS = 0$

1. $y_c = e^{mx}$
2. Plug y_c (and its derivatives) into the equation.
3. Factor/divide out e^{mx}.
4. Solve for m.
5. $y = y_c$

Nice $RHS = x^n, e^{ax}, \sin bx, \cos bx$

1. Find y_c.
2. Form the monster equation by applying the annihilator to both sides of the equation.
   $e^{ax} \to (D - a)$
   $\sin bx, \cos bx \to (D^2 + b^2)$
   $e^{ax} \sin bx, e^{ax} \cos bx \to ((D - a)^2 + b^2)$
   $x^n\ f(x) \to A^{n+1}$
3. Solve the monster equation (it is homogeneous) for y_m.
4. $y_p =$ all the terms from y_m that do not appear in y_c.
5. Plug y_p into the original equation and solve for $A, B, \ldots$
6. $y = y_c + y_p$

Not-so-Nice Variation of parameters.

1. Find $y_c = c_1v_1 + c_2v_2$.
2. $y_p = u_1v_1 + u_2v_2$
3. Plug y_p (and its derivatives) into the equation:
   $u_1'y_1 + u_2'y_2 = 0$
   $u_1'y_1' + u_2'y_2' = RHS$
4. Solve for v_1 and v_2.
5. Rewrite $y = y_c + y_p$

Anything LaPlace transforms.

1. Apply the LaPlace transform to both sides of the equation. Note the table of transforms at the end of the book.
2. Solve for $Y(s)$. Note: some books use $F(s)$.
3. Decompose the right-hand-side into partial fractions.
4. Apply the inverse LaPlace transform to both sides of the equation.

2 Reduction of Order

Given one solution, $y_1 \dots$

1. Let $y = u\ y_1$
2. Plug y (and its derivatives) into the equation.
3. All terms involving u (with no derivative) should cancel.
4. Let $v = \frac{du}{dx}$
5. Solve the (first order) equation for v.
6. Integrate to solve for u.
7. Rewrite $y = u\ y_1$

3 Variable Coefficients

Cauchy-Euler $ax^2\ \frac{d^2y}{dx^2} + bx\ \frac{dy}{dx} + c\ y = 0$ (degree matches derivative)

1. Let $y = x^\alpha$.
2. Plug u (and its derivatives) into the equation.
3. Divide by x^α.
4. Solve for α.
5. Rewrite $y = cx^\alpha$.

Regular Point The leading coefficient is not zero.

1. $y = \sum\limits_{n=0}^{\infty} a_n x^n$
2. Plug y (and its derivatives) into the equation.
3. Reindex the sums as necessary to combine into a single sum.
4. Form a recurrence relation, and solve for the highest index of a.
5. Solve the recurrence relation for a_n.
6. Rewrite $y = \sum\limits_{n=0}^{\infty} a_n x^n$

Singular Point The leading coefficient may be zero.

1. $y = \sum\limits_{n=0}^{\infty} a_n x^{n+r}$
2. Plug y (and its derivatives) into the equation.
3. Reindex the sums as necessary to combine into a single sum.
4. Solve the indicial polynomial for r.
5. Form a recurrence relation, and solve for the highest index of a.
6. Solve the recurrence relation for a_n.
7. Rewrite $y = \sum\limits_{n=0}^{\infty} a_n x^{n+r}$