Newton's Law of Cooling / Heating.

$$\frac{dT}{dt} = k(T_m - T) \quad \cdots\cdots \quad T = T_m + Ce^{-kt}$$

t	T
0	73
$\frac{1}{2}$	221
1	302

$$73 = T_m + C$$

$$221 = T_m + Ce^{-k/2}$$

$$302 = T_m + Ce^{-k}$$

$T_m = ?$

(2) - (1) $\quad 148 = c(e^{-k/2} - 1)$

(3) - (1) $\quad 229 = c\underbrace{(e^{-k} - 1)} = \cancel{c(e^{-k/2} - 1)}^{148}(e^{-k/2} + 1)$

$229 = 148(e^{-k/2} + 1) \rightarrow$ Solve for $e^{-k/2}$

Plug into, solve for c_0

Plug into (1), $T_m =$

- Room $1700\ ft^3$, No CO.

- CO into 6% @ $0.6 \frac{ft^3}{min}$

- Out: $0.6 \frac{\cancel{ft^3}}{min}$

A = Amt of CO in room (ft^3)

$$\frac{dA}{dt}\left[\frac{ft^3\, CO}{min}\right] = \text{in} - \text{out}$$

$$\frac{dA}{dt} = 0.6 \frac{\cancel{ft^3\, Air}}{min} \cdot \frac{6\ ft^3\, CO}{100\ \cancel{ft^3\, Air}} - \frac{0.6\, ft^3_{Air}}{min}\ \frac{A \overset{CO}{\cancel{ft^3}}}{1700\ \underset{ft^3}{Air}}$$

$$\cancel{\frac{dA}{dt} = \frac{.36}{100} - \frac{.6}{1700} A}$$

$$\frac{dA}{dt} = .036 - .00035247\, A \qquad A(0) = 0$$

$$\frac{dA}{dt} = 225 + 20 - \frac{30}{250} A$$

$$\frac{dA}{dt} = 245 - \frac{3}{25} A$$

I.C. How much sugar is initially in tank?

$$\frac{dA}{245 - \frac{3}{25} A} = dt$$

$A(0) = 32$

- Vat contains 250 gal
- In $9 \frac{\text{tbsp}}{\text{gal}}$, conc, $25 \frac{\text{gal}}{\text{min}}$
- In $4 \frac{\text{tbsp}}{\text{gal}}$, $5 \frac{\text{gal}}{\text{min}}$
- Out $30 \frac{\text{gal}}{\text{min}}$

A = Amount of sugar (tbsp)

$$\frac{dA}{dt}\left(\frac{\text{tbsp}}{\text{min}}\right) = \underbrace{9\frac{\text{tabsp}}{\cancel{\text{gal}}}\cdot 25\frac{\cancel{\text{gal}}}{\text{min}} + \frac{4\,\text{tbsp}}{\cancel{\text{gal}}}\cdot\frac{5\,\cancel{\text{gal}}}{\text{min}}}_{\text{In}}$$

$$- \frac{30\,\text{gal}}{\text{min}} \cdot \frac{A\ \text{tbsp}}{(250-5t)\,\text{gal}}$$

$y = \$$ in account

I.C. $y(0) = 40\,000$

$$\frac{dy}{dt} = .03y - 3000$$

$$\int \frac{dy\ (.03)}{.03y - 3000} = \int dt \rightarrow$$

$$\frac{1}{.03} \ln|.03y - 3000| = t + c$$

$$e^{\ln|.03y - 3000|} = e^{.03t + c}$$

$$.03y - 3000 = c\,e^{.03t}$$

$$.03y = 3000 + c\,e^{.03t}$$

$y(0) = 40\,000$

$$y = 100\,000 + c\,e^{.03t}$$

$$40\,000 = 100\,000 + c \rightarrow c = -60\,000$$

$$y = 100\,000 - 60\,000\,\cancel{\phi}e^{.03t}$$

$t = 4$

$y = \$32,350.19$

	t	y	Eqn $\frac{dy}{dt}$	step size dt	dy
I.C.	0	40 000	−1800	.4	$-720 = \frac{dy}{dt} \cdot dt$
	.4	39 280	−1821.6	.4	−728.64
	old t + dt	old y + dy			
	⋮				
	4				

$$\frac{dy}{dt} = .03y - 3000 = -1800$$

$S \leftarrow H$

$L \leftarrow B$

$E \leftarrow NE$

S) $\frac{dy}{dx} = x \sin(y) \rightarrow \int \frac{dy}{\sin(y)} = \int x \, dx$

L) $\frac{dy}{dx} + Py = Q$ IF: $\mu = e^{\int P}$

$e^{x^2}\left(\frac{dy}{dx} + 2x\,y = \sin(x)\, e^{-x^2}\right)$ $\mu = e^{\int 2x} = e^{x^2}$

$e^{x^2} \cdot \boxed{\frac{dy}{dx}} + 2xe^{x^2}\, y = \sin(x)$

$u \quad v' + \quad u' \quad v$

$\int (e^{x^2} y)' = \sin(x)$

$e^{x^2} y = -\cos(x) + C$

$y = -e^{-x^2}\cos(x) + ce^{-x^2}$

E) $(2x + y)\,dx + (x + 3y^2)\,dy = 0$

$$\frac{d}{dx}\left[f(x,y) = c\right]$$

$$\frac{\partial f}{\partial x} + \frac{\partial f}{\partial y}\frac{dy}{dx} = 0$$

$$\frac{\partial f}{\partial x}\,dx + \frac{\partial f}{\partial y}\,dy = 0$$

Check for exactness:

$$\frac{\partial}{\partial y}(2x+y) \stackrel{?}{=} \frac{\partial}{\partial x}(x+3y^2)$$

$$1 = 1 \quad \checkmark$$

$$\frac{\partial f}{\partial x} = 2x + y$$

$$f = x^2 + \boxed{xy} + c_1(y)$$

$$\int\left(\frac{\partial f}{\partial y} = x + 3y^2\right) dy$$

$$f = \boxed{xy} + y^3 + c_2(x)$$

$$f = xy + x^2 + y^3$$

Solution $xy + x^2 + y^3 = c$

H) $(x^2 - xy)\,dx + \cancel{(y^2 + 2xy)}\; y^2\,dy = 0$

All degree 2.

$x = uy$ or $y = ux$ ← dy has simpler coeff.

$$dy = u\,dx + x\,du$$

$$(x^2 - xux)\,dx + (ux)^2(u\,dx + x\,du) = 0$$

separable ...

Remember to substitute back.

$$u = \frac{y}{x}$$

B) $\frac{dy}{dx} + Py = Qy^n$ $\qquad u = y^{1-n}$

$\vdots$

Becomes linear

$-2y^{-3}\left[\frac{dy}{dx} + \frac{3}{x}y = 2x\,y^{3}\right]$ $\qquad u = y^{-2}$

$\frac{du}{dx} = -2y^{-3} \cdot \frac{dy}{dx}$

$\frac{du}{dx} - \frac{6}{x}u = -4x$ $\rightarrow$ Linear

$\mu = e^{\int -\frac{6}{x}} \ldots$

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$$\underbrace{\ }_{N} dy + \underbrace{(2xy - 4e^{-x^2})}_{M} dx = 0$$

Exact? $\frac{\partial}{\partial x}(1) \stackrel{?}{=} \frac{\partial}{\partial y}(2xy - 4e^{-x^2})$

$$0 \stackrel{?}{=} 2x \quad \text{No.}$$

$$\cancel{\frac{N_x - M_y}{M} = y's}$$

$$\cancel{\frac{0 - 2x}{2xy - 4e^{-x^2}}}$$

$$\frac{M_y - N_x}{N} = x's$$

$$\frac{2x - 0}{1} = 2x$$

$$IF = e^{\int 2x} = e^{x^2}$$