Physical Scenario $\longrightarrow$ Equation $\xrightarrow{*}$ Solution $\longrightarrow$ Interpret

- $\Sigma F = ma \longrightarrow mg - kx - p\cancel{x'} = m\cancel{a}$
- weight $\rightarrow$
- spring $\rightarrow$ Hooke's
- Drag $\rightarrow$ viscous vs nonviscous

LaPlace DE. $\xrightarrow{\mathcal{L}}$ Algebraic Equation $\longrightarrow$ $\boxed{Y =}$ $\xrightarrow{\mathcal{L}^{-1}}$ Sol. to DE

- Definition $\rightarrow$ examples $\mathcal{L}(t, \sin bt, e^{at})$
- Properties $\rightarrow$ more complicated examples [should be able to transform "Anything"]
- Solve DE's. $e^t y'' + \sin(t) y' - t^2 y = t+1$

$$(3+-4x)y'' + (-1x)y' + 3y = 0$$

$$y = \sum_{n=0}^{\infty} a_n x^n$$

$$y' = \sum_{n=0,1}^{\infty} n a_n x^{n-1}$$

$$y'' = \sum_{n=0,1,2}^{\infty} n(n-1) a_n x^{n-2}$$

$$(3-4x)\sum_{n=0,1,2}^{\infty} n(n-1)a_n x^{n-2} - 1x \sum_{n=0,1}^{\infty} n a_n x^{n-1} + 3\sum_{n=0}^{\infty} a_n x^n = 0$$

$$3\sum_{n=0,1,2}^{\infty} n(n-1)a_n x^{n-2} - 4\boxed{x}\sum_{n=0,1,2}^{\infty} n(n-1)a_n \boxed{x^{n-2}} - \boxed{x}\sum_{n=0,1}^{\infty} n a_n \boxed{x^{n-1}} + 3\sum_{n=0}^{\infty} a_n x^n = 0$$

x^{n-1} $\quad x^n$

$$\sum_{n=0,1,2}^{\infty} 3n(n-1)a_n x^{n-2} + \sum_{n=0,1,2}^{\infty} -4n(n-1)a_n x^{n-1} + \sum_{n=0,1}^{\infty} -n a_n x^n + \sum_{n=0}^{\infty} 3a_n x^n = 0$$

2 $\quad$ 2

1) Must have same power of x. $[x^n]$

2) Need same index of summation

$$\sum_{\cancel{n=-2,-1,0}}^{\infty} 3(n+2)(n+1)a_{n+2} x^n + \sum_{\cancel{n=-1,0,1}}^{\infty} -4(n+1)n\, a_{n+1} x^n + \sum_{\cancel{n=0,1}}^{\infty} -n a_n x^n + \sum_{n=0}^{\infty} 3a_n x^n = 0$$

$n=0 \quad n=0 \quad n=0$

$$\sum_{n=0}^{\infty}\left[3(n+2)(n+1)a_{n+2} - 4(n+1)n\,a_{n+1} \underbrace{- n a_n + 3a_n}_{+(3-n)a_n}\right] x^n = 0$$

$$(5+x)y'' + (1+x)y' + (-5+x)y = 0 \qquad y(0) = 4 \qquad y'(0) = 2$$

$10a_2 + 30a_3 x + 60a_4 x^2 + 2a_2 x + 6a_3 x^2 + 12a_4 x^3 + \dots$

$+a_1 + 2a_2 x + 3a_3 x^2 + 4a_4 x^3 + a_1 x + 2a_2 x^2 + 3a_3 x^3 + 4a_4 x^4 + \dots$

$a_0 = y(0) = 4$

$a_1 = y'(0) = 2$

$a_2 =$

$a_3 =$

$a_4 =$

Cauchy Product

$y = a_0 + a_1 x + a_2 x^2 + a_3 x^3 + a_4 x^4 + \dots$

$y' = a_1 + 2a_2 x + 3a_3 x^2 + 4a_4 x^3 + \dots$

$y'' = 2a_2 + 6a_3 x + 12a_4 x^2 + \dots$

$$(5+x)y'' + (1+x)y' + (-5+x)y = 0$$

x^0: $10a_2 + 1a_1 - 5a_0 = 0 \rightarrow$ Know a_2

x^1: $30a_3 + 2a_2 + 2a_2 + a_1 - 5a_1 + a_0 = 0 \rightarrow$ Know a_3

x^2: $60a_4 + 6a_3 + 3a_3 + 2a_2 - 5a_2 + a_1 = 0 \rightarrow$ Know a_4

$$e^x = \sum \frac{x^n}{n!} \qquad \cos(2x) = 1 - \frac{(2x)^2}{2!} + \cancel{\frac{(2x)^4}{4!}}$$

$$\sin x = \sum (-1)^n \frac{x^{2n+1}}{(2n+1)!}$$

Cauchy-Euler

$$x^2 y'' - 6xy' + 12y = 0 \qquad y(1) = 3 \qquad y'(1) = 7$$

Try $y = x^\alpha$ (Ansatz)

$y' = \alpha x^{\alpha-1}$

$y'' = \alpha(\alpha-1)x^{\alpha-2}$

$\alpha(\alpha-1)x^\alpha - 6\alpha x^\alpha + 12x^\alpha = 0$

$x^\alpha\left[\alpha^2 - \alpha - 6\alpha + 12\right] = 0$

$\alpha^2 - 7\alpha + 12 = 0$

$(\alpha-3)(\alpha-4) = 0$

$\alpha = 3 \quad \alpha = 4$

$$y = c_1 x^3 + c_2 x^4$$