$y'''' + 18y'' + 81y = 0$ $\qquad y^{(4)}$ — Short-hand for y''''

$m^4 + 18m^2 + 81 = 0$ $\qquad$ A, M $\qquad y = ce^{mx}$

$(m^2+9)(m^2+9) = 0$

$\boxed{\text{Recall: } m = a \pm bi \qquad y = c_1 e^{ax}\cos bx + c_2 e^{ax}\sin bx}$

$m^2 + 9 = 0 \qquad m^2+9=0$

$m^2 = -9$

$m = \pm 3i$ $\quad$ multiplicity 2

$a = 0$

$b = 3$

$y = c_1 e^{0x}\cos 3x + c_2 e^{0x}\sin 3x + c_3 x e^{0x}\cos 3x + c_4 x e^{0x}\sin 3x$

$y = c_1 \cos(3x) + c_2\sin(3x) + c_3 x\cos(3x) + c_4 x\sin(3x)$

$y(0) = -3 \qquad -3 = c_1$

$y'(0) = -7 \qquad y' = -3c_1 \cancel{\sin(3x)} + 3c_2\cos(3x) + c_3[1\cdot\cos(3x) + \cancel{x\cdot -3\sin 3x}]$
$+ c_4[1\cdot\cancel{\sin(3x)} + \cancel{x\cdot 3\cos(3x)}]$

$y''(0) = 21$

$y'''(0) = 135$

$y''' - 5y'' - 25y' + 125y = 0$

$m^3 - 5m^2 - 25m + 125 = 0$

$1 : -5 \qquad 1 : -5$

$m^2(m-5) - 25(m-5) = 0$

$(m-5)(m^2-25) = 0$

$(m-5)(m-5)(m+5) = 0$

$y = c_1 e^{5x} + c_2 x e^{5x} + c_3 e^{-5x}$

$y(0) = -4 \qquad y'(0) = -13 \qquad y''(0) = -130$

$y: \quad -4 = c_1 + c_2(0) + c_3$

$y' = 5c_1 e^{5x} + c_2 [5xe^{5x} + e^{5x}] - 5c_3 e^{-5x}$

$-13 = 5c_1 + c_2[0+1] - 5c_3$

$y'' = 25c_1 e^{5x} + c_2 [25xe^{5x} + 5e^{5x} + 5e^{5x}] + 25c_3 e^{-5x}$

$(x \cdot e^{ax})' = x \cdot ae^{ax} + 1 \cdot e^{ax} = axe^{ax} + e^{ax}$

$$y = c_1 e^{3t} + c_2 e^{-5t}$$

$y(0) = \alpha$

as $t \to \infty$, $y \to 0$

as $t \to \infty$,

$e^{3t} \to \infty$ [So $c_1 = 0$]

$e^{-5t} \to 0$

Recall: $\lim_{t \to \infty} \frac{c}{t} = 0$

$\lim_{t \to \infty} e^{t} = \infty$

$\lim_{t \to \infty} e^{-t} = \lim_{t \to \infty} \frac{1}{e^{t}} = \frac{1}{\infty} \to 0$

$$y = c_2 e^{-5t}$$

$$\alpha = c_2$$

$y''' + 7y'' - 5y' - 75y = 0$

No "obvious" factors.

$m^3 + 7m^2 - 5m - 75 = 0$

Rational Zeros Test:

$px^n + \dots + q = 0$

only possible rational zeros are of the form

$x = \pm \dfrac{Q}{P}$

where Q, P are factors of q, p.

only possible rational zeros:

$m = \pm \dfrac{1, 3, 5, 15, 25, 75}{1}$

$m = 1, -1, 3, -3, 5, -5, 15, -15, 25, -25, 75, -75$

Trial & error:

TI-83: $\boxed{Y_1} = X^3 + 7X^2 - 5X - 75$

$\boxed{2^{nd}}$ $\boxed{\text{Tableset}}$ → Independent: Ask

$\boxed{2^{nd}}$ $\boxed{\text{Table}}$ manually enter the possibilities

$m = 3$ works (it gives $= 0$)

factor?

Quad form?

$(m-3)(m^2 + 10m + 25) = 0$

	m^3	m^2	m^1	m^0
3	1	7	-5	-75
	↓	+	+	+
		3	30	75
	1	10	25	0
	m^2	m^1	m^0	remainder

Pascal's Triangle

- First, last entry in a row are 1.
- Middle entries = sum of left & right from previous row.

row 0   1
row 1   1   1
row 2   1   2   1
3   1   3   3   1
4   1   4   6   4   1

- Binomial Theorem $(x+y)^n = C_n x^n y^0 + C_{n-1} x^{n-1} y^1 + C_{n-2} x^{n-2} y^2 \cdots + x^1 y^{n-1} + x^0 y^n$

coefficients from Pascal's triangle.

Ex: $(x+y)^4 = 1x^4y^0 + 4x^3y^1 + 6x^2y^2 + 4x^1y^3 + 1x^0y^4$

- Liebnitz Rule (Products)

$$(u\cdot v)^{(4)} = 1u^{(4)}v + 4u^{(3)}v' + 6u''v'' + 4u'v^{(3)} + 1uv^{(4)}$$

Ex: $y = ce^{4x}\sin(3x)$ $y''''(0) = 12$

$$y'''' = c\left[1\cdot 4^4 e^{4x}\sin(3x) + 4\cdot 4^3 e^{4x}\cdot 3\cos(3x) + 6\cdot 4^2 e^{4x}\cdot -9\sin(3x) + 4\cdot 4e^{4x}\cdot -27\cos(3x) + 1e^{4x}\cdot 81\sin(3x)\right]$$

$$m^3 + 6m^2 + 12m + 8 = 0$$

$$m^3 + 3m^2\cdot 2^1 + 3\cdot m^1\cdot 2^2 + 1m^0\cdot 2^3 = 0$$

$$(m+2)^3 = 0$$

$$i^0 = i^{4k} = 1 \qquad i^2 = i^{4k+2} = -1$$

$$i^1 = i^{4k+1} = i \qquad i^3 = i^{4k+3} = -i$$

Euler's Formula (Bonus)

Recall

$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}$$

$$\sin x = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)!}$$

$$\cos x = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n)!}$$

$$e^{it} = \sum_{n=0}^{\infty} \frac{(it)^n}{n!} = \sum_{n=0}^{\infty} \frac{i^n t^n}{n!}$$

$$= 1 + \frac{it}{1!} - \frac{t^2}{2!} - \frac{it^3}{3!} + \frac{t^4}{4!} + \frac{it^5}{5!} - \frac{t^6}{6!} - \frac{it^7}{7!} + \cdots$$

$$= \sum_{n=0}^{\infty} \frac{(-1)^n t^{2n}}{(2n)!} + i \sum_{n=0}^{\infty} \frac{(-1)^n t^{2n+1}}{(2n+1)!}$$

$$= \cos(t) + i\sin(t)$$