$y' - 4y = 0$ $\qquad\qquad$ $y = \sum_{n=0}^{\infty} a_n x^n$

$\sum_{n=0,1}^{\infty} a_n n x^{n-1} \xrightarrow{+1} - 4\sum_{n=0}^{\infty} a_n x^n = 0$ $\qquad\qquad$ $y' = \sum_{n=0,1}^{\infty} a_n n x^{n-1}$

$1 \downarrow$

$\sum_{\cancel{n=-1}\ \cancel{0}}^{\infty} a_{n+1}(n+1) x^n - 4\sum_{n=0}^{\infty} a_n x^n = 0$

$n=0$

- Must have like terms.
- Must have same index.

$$\sum_{n=0}^{\infty} \left[a_{n+1}(n+1) - 4a_n \right] x^n = 0$$

Every coeff $= 0$

$$a_{n+1}(n+1) - 4a_n = 0$$

$$a_{n+1}(n+1) = 4a_n$$

$$\boxed{a_{n+1} = \frac{4a_n}{n+1}} \quad \text{for } n = 0, 1, 2, \ldots$$

$n=0$ $\quad a_1 = \frac{4a_0}{1} = \frac{4^1 a_0}{1!}$

$n=1$ $\quad a_2 = \frac{4a_1}{2} = \frac{4 \cdot 4a_0}{2 \cdot 1} = \frac{4^2 a_0}{2!}$

$n=2$ $\quad a_3 = \frac{4a_2}{3} = \frac{4 \cdot 4 \cdot 4a_0}{3 \cdot 2 \cdot 1} = \frac{4^3 a_0}{3 \cdot 2 \cdot 1} = \frac{4^3 a_0}{3!}$

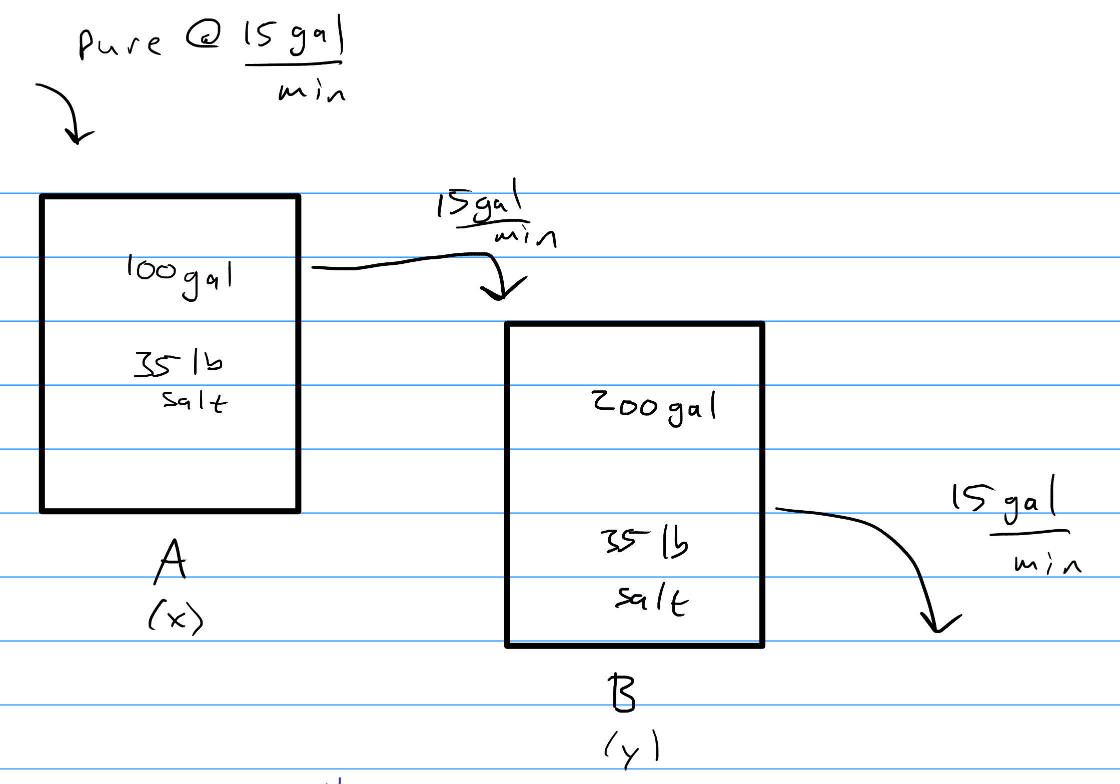
$n=3$ $\quad a_4 = \frac{4a_3}{4} = \frac{4 \cdot 4^3 a_0}{4 \cdot 3 \cdot 2 \cdot 1} = \frac{4^4}{4!} a_0$

$a_n = \frac{4^n}{n!} a_0$ $\qquad\qquad$ $a_{n+1} = \frac{4^{n+1}}{(n+1)!} a_0$

Recall: $y = \sum_{n=0}^{\infty} a_n x^n$

$$y = \sum_{n=0}^{\infty} \frac{4^n}{n!} a_0 x^n = a_0 \sum_{n=0}^{\infty} \frac{4^n x^n}{n!} = a_0 e^{4x}$$

$$y = c e^{4x}$$



$\frac{lbs}{min}$ $\quad \frac{dx}{dt} = 15\frac{gal}{min}\cdot\frac{0\ lb}{1\ gal} - 15\frac{gal}{min}\cdot\frac{x\ lb}{100\ gal}$

$$\frac{dx}{dt} = -\frac{15}{100}x + 0y$$

$$\frac{dy}{dt} = +15\frac{gal}{min}\cdot\frac{x\ lbs}{100\ gal} - 15\frac{gal}{min}\cdot\frac{y\ lbs}{200\ gal}$$

$$\frac{dy}{dt} = \frac{15}{100}x - \frac{15}{200}y$$

$$\begin{bmatrix} x \\ y \end{bmatrix}' = \begin{bmatrix} -\frac{15}{100} & 0 \\ \frac{15}{100} & -\frac{15}{200} \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} \qquad \vec{X}' = A\vec{X}$$

$$A = \begin{bmatrix} -3/20 & 0 \\ 3/20 & -\frac{3}{40} \end{bmatrix} \qquad \det(A-\lambda I) = \cancel{\det}\begin{bmatrix} -3/20-\lambda & 0 \\ \frac{3}{20} & -\frac{3}{40}-\lambda \end{bmatrix} = 0$$

$$= \left(-\frac{3}{20}-\lambda\right)\left(-\frac{3}{40}-\lambda\right) - 0$$

$$= \left(-\frac{3}{20}-\lambda\right)\left(-\frac{3}{40}-\lambda\right) \underset{\text{set}}{=} 0$$

$$\lambda_1 = -\frac{3}{20} \qquad \lambda_2 = -\frac{3}{40}$$

$\lambda_1 = -3/20$

$$\left[\begin{array}{cc|c} -\frac{3}{20} - -3/20 & 0 & 0 \\ 3/20 & -\frac{3}{40} - -\frac{3}{20} & 0 \end{array}\right]$$

$$\sim \left[\begin{array}{cc|c} 0 & 0 & 0 \\ 3/20 & 3/40 & 0 \end{array}\right] \sim \frac{20}{3}\cdot\left[\begin{array}{cc|c} 3/20 & 3/40 & 0 \\ 0 & 0 & 0 \end{array}\right] \sim \left[\begin{array}{cc|c} 1 & \frac{1}{2} & 0 \\ 0 & 0 & 0 \end{array}\right]$$

y free

$x + \frac{1}{2}y = 0 \qquad y = t$

$x = -\frac{1}{2}t$

$$\vec{X}_1 = \begin{bmatrix} -\frac{1}{2}t \\ t \end{bmatrix}$$

@ $t = -2$

$$\vec{X}_1 = \begin{bmatrix} 1 \\ -2 \end{bmatrix}$$

Ideally: No fractions
No common factor
No lead -

$\lambda_2 = -\frac{3}{40}$

$$\left[\begin{array}{cc|c} -3/20 - -3/40 & 0 & 0 \\ 3/20 & -\frac{3}{40} - -\frac{3}{40} & 0 \end{array}\right] \sim -\frac{40}{3}\left[\begin{array}{cc|c} -3/40 & 0 & 0 \\ 3/20 & 0 & 0 \end{array}\right] \sim \left[\begin{array}{cc|c} 1 & 0 & 0 \\ 3/20 & 0 & 0 \end{array}\right]$$

$-\frac{3}{20}R_1 + R_2 \to R_2$

$$\sim \left[\begin{array}{cc|c} 1 & 0 & 0 \\ 0 & 0 & 0 \end{array}\right]$$

y free

$1x = 0$
$y = t$

$$\vec{X}_2 = \begin{bmatrix} 0 \\ t \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$\boxed{\vec{X} = c_1\vec{X}_1 e^{\lambda_1 t} + c_2\vec{X}_2 e^{\lambda_2 t}}$$

$$\vec{X} = c_1\begin{bmatrix} 1 \\ -2 \end{bmatrix}e^{-\frac{3}{20}t} + c_2\begin{bmatrix} 0 \\ 1 \end{bmatrix}e^{-3/40\,t}$$

I.C. $x(0) = 35$
$y(0) = 35$

$$\vec{X}(0) = \begin{bmatrix} 35 \\ 35 \end{bmatrix}$$

t=0

$$\begin{bmatrix} 35 \\ 35 \end{bmatrix} = c_1\begin{bmatrix} 1 \\ -2 \end{bmatrix} + c_2\begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$\hat{i}$: $35 = c_1 + 0 \to c_1 = 35$

$\hat{j}$: $35 = -2c_1 + c_2 \to 35 = -70 + c_2$

$105 = c_2$

$$\vec{X} = 35\begin{bmatrix} 1 \\ -2 \end{bmatrix}e^{-3/20\,t} + 105\begin{bmatrix} 0 \\ 1 \end{bmatrix}e^{-3/40\,t}$$

$$x = 35e^{-3/20\,t}$$

$$y = -70e^{-3/20\,t} + 105e^{-3/40\,t}$$