$$\int \frac{3x^2-7x+2}{(x-1)(x^2+4)}\,dx \qquad \left[\frac{3x^2-7x+2}{(x-1)(x^2+4)} = \frac{A}{x-1} + \frac{Bx+C}{x^2+4}\right] \text{ original denom.}$$

$$3x^2-7x+2 = A(x^2+4) + (Bx+C)(x-1)$$

Convenient Values: $x=1$ (Both Sides)

$$-2 = 5A \Rightarrow A = -2/5$$

Matching Coefficients:

$$\left.\begin{aligned} x^2&: \; 3 = A+B \\ x^1&: \; -7 = -B+C \\ x^0&: \; 2 = 4A-C \end{aligned}\right\} \text{ Solve the system.}$$

Case 1: (Distinct Linear Factors)

$$\frac{3x^2-1}{(x+2)(x-1)(x-5)} = \frac{A}{x+2} + \frac{B}{x-1} + \frac{C}{x-5} \qquad \left[\begin{array}{l}\text{3 convenient values} \\ x=-2, +1, +5\end{array}\right]$$

Case 2: (Distinct Irreducible Quadratic Factors)

$$\frac{x+4}{(x-1)(x^2+1)(x^2+4)} = \frac{A}{x-1} + \frac{Bx+C}{x^2+1} + \frac{Dx+E}{x^2+4} \qquad \left[\begin{array}{l}\text{1 convenient value} \\ x=+1\end{array}\right]$$

Case 3: (Repeated Linear Factors)

$$\frac{x^2+7x+2}{(x+1)(x-3)^2(x+4)^3} = \frac{A}{x+1} + \frac{B}{x-3} + \frac{C}{(x-3)^2} + \frac{D}{x+4} + \frac{E}{(x+4)^2} + \frac{F}{(x+4)^3}$$

$$\left[\begin{array}{l}\text{3 convenient values} \\ x=-1, +3, -4\end{array}\right] \qquad \frac{A}{x+1} + \frac{Bx+C}{(x-3)^2} + \frac{Dx^2+Ex+F}{(x+4)^3}$$

$$B(x-3+3)+C$$

$$\frac{B(x-3)}{(x-3)^2} + \frac{3B+C}{(x-3)^2}$$

Case 4: (Repeated Irreducible Quadratic Factors)

$$\frac{2x+3}{(x^2+1)^2(x^2+4)^3} = \frac{Ax+B}{x^2+1} + \frac{Cx+D}{(x^2+1)^2} + \frac{Ex+F}{x^2+4} + \frac{Gx+H}{(x^2+4)^2} + \frac{Ix+J}{(x^2+4)^3}$$

[no convenient values]

Ex:
$$\frac{s+3}{s(s^2+1)} = \frac{A}{s} + \frac{Bs+C}{s^2+1} = \frac{3}{s} + \frac{-3s+1}{s^2+1} = \frac{3}{s} - \frac{3s}{s^2+1} + \frac{1}{s^2+1}$$

$$s+3 = A(s^2+1) + (Bs+C)s$$

$s=0$: $3 = A$

s^2: $A + B = 0 \rightarrow B = -3$

s^1: $C = 1$

s^0: $A = 3$

given matrix $A = \begin{bmatrix} A_{11} & A_{12} & \cdots & A_{1n} \\ \vdots & & & \vdots \\ A_{n1} & A_{n2} & \cdots & A_{nn}\end{bmatrix} = \left[A_{\text{row, col}}\right]$

Want to find a vector $\vec{x}$, so that $A\vec{x} = \lambda\vec{x}$,
- λ is an eigenvalue for A,
- $\vec{x}$ is its eigenvector.

λ: scalar

Ex: Rotate the earth, the axis is fixed.

$A\vec{x} = 1\vec{x}$

$\vec{x}$: Vector pointing from pole to pole

How? $A\vec{x} = \lambda\vec{x}$ (solution is <u>NOT</u> unique)

$A\vec{x} - \lambda\vec{x} = \vec{0} \longrightarrow A\vec{x} - \lambda I\vec{x} = 0$

$(A-\lambda)\vec{x} = \vec{0}$? ↳ what is this? $\qquad (A-\lambda I)\vec{x} = 0$

Need $\det(A-\lambda I) = 0$. Characteristic equation

$$\boxed{\begin{aligned}&\det(A-\lambda I) = 0 \\ &(A-\lambda I)\vec{x} = 0\end{aligned}}$$

Ex: $A = \begin{bmatrix} 2 & 4 \\ 4 & 2\end{bmatrix}$

1) $\det\begin{bmatrix} 2-\lambda & 4 \\ 4 & 2-\lambda\end{bmatrix} \overset{\text{set}}{=} 0$

$(2-\lambda)^2 - 16 = 0$

$(2-\lambda)^2 = 16$

$2-\lambda = -4$	$2-\lambda = 4$
$-\lambda = -6$	$-\lambda = 2$
$\lambda = 6$	$\lambda = -2$

2) $\lambda = 6$

$$-\tfrac{1}{4}\begin{array}{c}\begin{array}{ccc} x_1 & x_2 & 0\end{array} \\ \left[\begin{array}{cc|c} -4 & 4 & 0 \\ 4 & -4 & 0\end{array}\right]\end{array} \sim \left[\begin{array}{cc|c} 1 & -1 & 0 \\ 4 & -4 & 0\end{array}\right] \cdot -4$$

$$\sim \left[\begin{array}{cc|c} 1 & -1 & 0 \\ 0 & 0 & 0\end{array}\right]$$

x_2 is free (no leading 1 in 2^{nd} column)

$x_2 = t \qquad x_1 - t = 0$
$\qquad\qquad x_1 = t \qquad \vec{x} = \begin{bmatrix} t \\ t\end{bmatrix}$

choose $t=1 \rightarrow \vec{x} = \begin{bmatrix} 1 \\ 1\end{bmatrix}$

Note: Choose t so that
- No fractions.
- No common factors.
- First non-zero entry is positive.

Leading 1 - first non zero entry in a row.

leading

$\lambda = -2$: $\tfrac{1}{4}\left[\begin{array}{cc|c} 4 & 4 & 0 \\ 4 & 4 & 0\end{array}\right] \sim \left[\begin{array}{cc|c} 1 & 1 & 0 \\ 0 & 0 & 0\end{array}\right]$

$1x_1 + 1x_2 = 0 \qquad x_2 = t$ (no leading 1 in x_2)

$x_1 = -t$

$\vec{x} = \begin{bmatrix} -t \\ t\end{bmatrix} = \begin{bmatrix} 1 \\ -1\end{bmatrix}$

$t = -1$