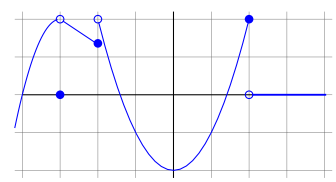
Math 180 Review

updated July 3, 2013

Let $f(x) = \begin{cases} \frac{-1}{x+3}, & x < -3 \\ x-1, & x = -3 \\ \frac{1}{x+3}, & -3 < x \leq -2 \\ x^2 - 3, & -2 < x \leq 2 \\ x-1, & x > 2 \end{cases}$

Define $g(x)$ by



1. $\lim_{x \to -3} f(x)$
2. $\lim_{x \to -2} f(x)$
3. $\lim_{x \to 2} f(x)$
4. $\lim_{x \to 3} f(x)$
5. $\lim_{x \to -3} g(x)$
6. $\lim_{x \to -2} g(x)$
7. $\lim_{x \to 2} g(x)$
8. $\lim_{x \to 3} g(x)$
9. $\lim_{x \to 1} \frac{x}{x+1}$
10. $\lim_{x \to 1} \frac{x}{1-x}$
11. $\lim_{x \to 1} \frac{x}{(1-x)^2}$
12. $\lim_{x \to \infty} \frac{x^2-x+3}{x^2-3}$
13. $\lim_{x \to 2} \frac{x^3-8}{x-2}$
14. $\lim_{x \to 9} \frac{\sqrt{x}-3}{x-9}$
15. $\lim_{x \to 0} \frac{e^{3x}-1}{2x}$
16. $\lim_{x \to 0} \csc x - \frac{1}{x}$
17. $\lim_{x \to 0} \frac{e^{3x^2}-1}{x^2}$
18. $\lim_{x \to 0} \frac{x - \sin x}{x^3}$
19. $\lim_{x \to 0^+} \sqrt[4]{x} \ln x^6$
20. $\lim_{x \to \frac{\pi}{2}} \cos^{1-\sin x} x$
21. $\lim_{x \to \infty} \left(1 - \frac{3}{x} + \frac{7}{x^2}\right)^x$
22. $\lim_{x \to \infty} 2x^{\frac{2}{x}}$
23. $\lim_{x \to 0} \sqrt[3]{x} \cos\left(e^{\frac{1}{x}}\right)$
24. $\lim_{x \to -\infty} \frac{\sqrt{4x^2+5}}{3x}$

Determine whether or not each function is continuous at the given point.

25. $g_1(x) = \frac{x^2-1}{x-1}$ at $x = 1$
26. $g_2(x) = \begin{cases} 2x-3, & x < 3 \\ 3x-2, & x \geq 3 \end{cases}$ at $x = 3$
27. $g_3(x) = \begin{cases} x^2-1, & x \neq 2 \\ 4, & x = 2 \end{cases}$ at $x = 2$
28. $g_4(x) = \frac{x+2}{x-1}$ at $x = 4$

Find any points of discontinuity and determine if each is removeable.

29. $f_1(x) = \frac{\sin 2x}{x}$
30. $f_2(x) = \begin{cases} x^2+1, & x \leq 2 \\ 2x-1, & x > 2 \end{cases}$
31. $f_3(x) = \frac{x}{x^2+4}$

Determine whether or not the function has a zero in the indicated interval.

32. $f(x) = x^3 - 5x + 2$ on $[0, 1]$
33. $g(x) = x^3 + x - 3$ on $[0, 1]$
34. $h(x)x^3 - x^2 - 2x + 1$ on $[0, 2]$

Given ϵ, find a δ so that the definition of limit is satisfied.

35. $\lim_{x \to 2} 3x - 1 = 5$, $\epsilon = 0.1$
36. $\lim_{x \to 1} 2 - 3x = -1$, $\epsilon = 0.01$
37. $\lim_{x \to 2} x^2 = 4$, $\epsilon = 0.03$

Prove the given limit.

38. $\lim_{x \to -2} 2x + 5 = 1$
39. $\lim_{x \to 1} 4 - 2x = 2$
40. $\lim_{x \to 3} x^2 = 9$

Compute each derivative.

41. $x^2 \sin 3x$

42. $\frac{\arctan 2x}{x^3}$

43. $\ln(x^2 - 1)$

44. $\sqrt{x}e^{-3x}$

45. $\frac{\cos x}{\arcsin x}$

46. $\tan(\ln x)$

47. $\cos(x)e^{-2x}$

48. $\frac{\arcsin x}{\ln x}$

49. $\arctan e^{4x}$

50. $\ln(x)e^{5x}$

51. $\arctan e^{x^2}$

52. $\cos(x \arcsin x)$

53. $x \cos(x2^x)$

54. $\arcsin \frac{\cos x}{\ln x}$

55. $\frac{\arctan x}{e^{\sin x}}$

56. $\frac{x \tan x}{\ln x}$

57. $x \ln(x)e^x$

58. $\tan(\ln(x)e^x)$

59. $\arcsin \frac{\ln x}{x}$

60. $\ln(\cos(e^x))$

61. $\log_2\left(\log_7 x^2\right)$

62. $\log_3(e^{\log_7 x})$

63. $\sin\left(\ln\left(\cos\left(e^{7x}\right)\right)\right)$

64. $\sqrt{2x-3}\sqrt[3]{3x-2}(x-1)^7$

65. $\sqrt{\frac{x^2+1}{x-4}}\sqrt[3]{\frac{x-2}{x^2+4}}$

66. $2^{3x} \cdot 3^{4x}$

67. 2^{3^x}

68. x^x

69. $(\sin x)^x$

70. u^v, for general functions u and v.

Find the derivative by applying the limit definition.

71. $h_1(x) = x^3 - 7x + 2$

72. $h_2(x) = \sqrt{x+3}$

73. $h_3(x) = \frac{1}{x-2}$

74. $h_4(x) = \frac{1}{\sqrt{x}}$

Find the derivative at the indicated point by applying the alternate definition.

75. $\phi_1(x) = x^3 - 1$ at (2,7)

76. $\phi_2(x) = \sqrt{x-1}$ at (5,2)

77. $\phi_3(x) = \frac{1}{x+2}$ at (-1,1)

78. $\phi_4(x) = \frac{1}{\sqrt{x}}$ at (1,1)

Find the absolute extrema on the given interval.

79. $x^3 - 12x + 2$ on $[-3, 3]$

80. $x^3 - 3x^2 + 4x$ on $[1, 3]$

81. $x - 2\sqrt[3]{x^2}$ on $[-1, 5]$

Use the calculus-informed curve sketching techniques to graph the following.

82. $f(x) = x^3 - 3x^2 - 9x + 27$

83. $g(x) = \frac{x}{x^2-1}$

84. $h(x) = (x-1)^2(x-3)^3$

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85. For each function, determine if Rolle's theorem applies on $[-2, 2]$ and if so, find the value of c guaranteed by Rolle's theorem.
    (a) $f(x) = \frac{1}{x^2}$
    (b) $g(x) = x^2 - x + 1$
    (c) $h(x) = x^2 - 3$

86. For each function, determine if the mean value theorem applies on $[-2, 2]$ and if so, find the value of c guaranteed by the mean value theorem.
    (a) $f(x) = \frac{1}{x-1}$
    (b) $g(x) = x^2 - x + 1$
    (c) $h(x) = x - \sqrt[3]{x}$

87. Find the equation of the tangent line to $y = \sqrt{x-2}$ at $x = 6$.

88. Find the equation of the normal line to $y = \frac{1}{x^2}$ at $x = -2$.

89. A stone is thrown upward with an initial velocity of 11 ft/s from the roof of a building 27 ft high. Find...

(a) The maximum height the stone reaches.

(b) The time it takes for the stone to hit the ground.

(c) The velocity and speed of the stone when it hits the ground.

(d) The average velocity from $t = 0$ to $t = 1$.

90. Find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$, where $xy = x^2 - y^2$.

91. At 3:25, a car begins travelling north at 60 mph. Fifteen minutes later, a second car leaves the same spot, travelling east at 75 mph. At 4:00, find the rate at which the distance between the two cars is increasing.

92. A spherical balloon is being filled at 12 cc/s. Find its rate of change in surface area when its volume is 60 cc.

93. A rocket is fired from ground level with an acceleration of 6 ft/s^2, 50 ft from an observer on ground level. Find the rate at which the observer's angle of elevation is changing when the rocket is 75 ft from the ground.

94. 16 cm of wire is to be cut into two pieces. The first piece will be bent into the shape of a square, while the second will be bent to form an equilateral triangle. How should the wire be cut so that the enclosed area is maximum?

95. 24 in^2 of material is available to construct a square-bottomed box. Find the dimensions of the box that maximize its volume.

96. Find the point on the graph of $y = \sqrt{x}$ that is closest to $(0, 4)$.

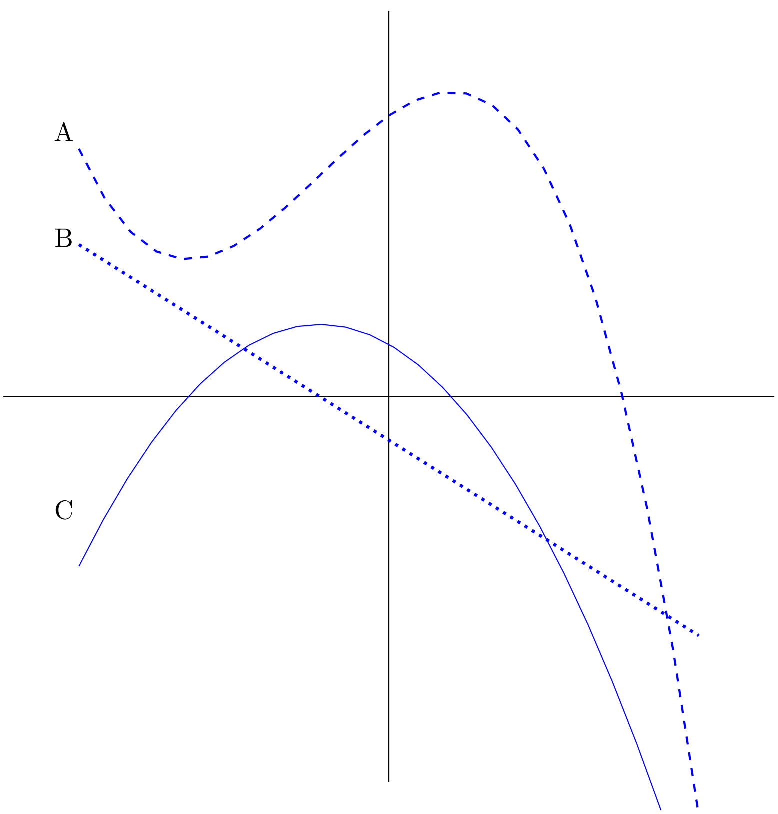
97. A can is to hold $350cm^3$. The material for the side of the can costs \$0.02 per cm^2, that of the bottom costs \$0.03 per cm^2, while the top costs \$0.04 per cm^2. Find the dimensions of the can that minimize the cost.

98. A movie screen is 30 ft high, and raised 8 ft off the floor. How far from the screen should you sit to maximize your viewing angle?

99. Use differentials to estimate $\sqrt{99}$.

100. A cube is measured to be $12.1 \pm 0.05cm$ on a side. Estimate the percent error in volume of this measurement.

101. Given the graphs labeled A, B and C, identify which is $f(x), f'(x), f''(x)$.



Evaluate each integral.

102. $\int 3x^5 - 6x^2 - 4x + 5\ dx$

103. $\int\limits_0^1 \frac{e^x+1}{e^x}\ dx$

104. $\int x \sec x^2\ dx$

105. $\int\limits_0^{\frac{\pi}{4}} (1 + \sec x)^2\ dx$

106. $\int 2x\sqrt{3x-1}\ dx$

107. $\int\limits_0^{\pi} \sin 3x \cos 3x\ dx$

108. $\int \frac{\sec 2x \tan 2x}{1+\sec 2x}\ dx$

109. $\int\limits_{\ln 2}^{\ln 3} \frac{e^{-x}}{\sqrt{1-e^{-2x}}}\ dx$

110. $\int \frac{2e^x}{\sqrt[3]{e^x+1}}\ dx$

111. $\int\limits_0^{\frac{\pi}{4}} \frac{\arctan x}{1+x^2}\ dx$

112. $\int \frac{x}{x^4+1}\ dx$

113. $\int\limits_{-5}^{5} |x^2 - x - 2|\ dx$

114. $\int \frac{x^2}{x^2+1}\ dx$

115. $\int\limits_0^1 x^3(x^2+1)^6\ dx$

116. $\int \frac{\csc^2 x}{2+\cot x}\ dx$

117. $\int\limits_0^2 5^{x/2}\ dx$

118. $\int \frac{1}{e^x+e^{-x}}\ dx$

119. $\int\limits_1^e \frac{1}{x(1+\ln x)^2}\ dx$

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120. Find the area bounded by $y = 3x^2 + 6x + 11$ on the interval $[0, 2]$.

121. Use a Riemann sum to find the area of the region bounded by $y = 4x - x^2$ and $y = 0$.

122. Suppose $f'''(x) = 12x - 6$, $f''(1) = -3$, $f'(2) = 1$ and $f(0) = 3$. Find $f(x)$.

123. Gravity on the moon's surface is 5.33 ft$/s^2$. Derive a formula for the motion of a rock dropped from a height of 4 ft.

124. Find the average value of $x^2 - 2x + 4$ on [0,3], and find the value of c guaranteed by the mean value theorem.

125. Compute $\frac{df}{dx}$, where $f(x) = \int\limits_{x^2}^{7} \sin e^t\ dt$.