MATH 165
FINAL EXAM REVIEW – Fall 2013

Conic Sections:

General Form:

A) $16x^2 - 9y^2 - 64x - 54y - 161 = 0$

B) $4x^2 + y^2 + 8x - 6y + 9 = 0$

C) $x^2 + 2x - y - 8 = 0$

D) $y^2 + 4x + 6y - 3 = 0$

E) $x - 6y + 14 = 0$

F) $x^2 + y^2 - 8y = 0$

Standard Form:

G) $(x + 1)^2 = (y + 9)$

H) $\frac{(x+1)^2}{1} + \frac{(y-3)^2}{4} = 1$

I) $y = \frac{1}{6}x + \frac{7}{3}$

J) $\frac{(x-2)^2}{9} - \frac{(y+3)^2}{16} = 1$

K) $x^2 + (y - 4)^2 = 16$

L) $(y + 3)^2 = -4(x - 3)$

1. Fill in the following chart with the equations above and the find the information in the last column.

conic section	**general form**	**standard form**	**find**
line			slope, x and y intercepts
circle			center, radius
parabola			vertex, focus, focal width, equation of directrix, AOS
parabola			vertex, focus, focal width, equation of directrix, AOS
ellipse			center, vertices, foci, major axis, minor axis
hyperbola (B = 0)			center, vertices, foci, transverse axis, conjugate axes, slopes of asymptotes

2. Find the equation of a parabola with vertex at (-1,4) and a focus at (2, 4).

3. Find the equation of parabola with a vertex (2, 5), a vertical axis of symmetry, and which passes through the point (4, -2).

4. Find the equation of an ellipse with a major vertical axis of 20 units, a minor axis of 16 units and a center at (3, -4).

5. Find the equation of the hyperbola with vertices at (5, 0), and (–5, 0) and foci: (13, 0), (–13, 0). Write the equation in general form.

6. Find the equation of the circle passing through (8, –2) and with center (–2, 4). Leave the equation in standard form.

Limits: Find the indicated limit of each of the following:

1. $\lim_{x\to 1} \frac{3}{x^2+2}$

2. $\lim_{x\to 0^+} \frac{2}{x}$

3. $\lim_{x\to 3} \frac{x^2-4x+3}{x-3}$

4. $\lim_{\Delta x\to 0} \frac{(x+\Delta x)^2-x^2}{\Delta x}$

5. $\lim_{x\to\infty} \frac{5x-x^2}{2x^2-3x}$

6. $\lim_{x\to\infty} \frac{7x+3}{2x^2-1}$

7. $\lim_{x\to\infty} \frac{6}{x^2-4x+3}$

8. $\lim_{x\to 4^+} \frac{x+2}{4-x}$

9. Find the instantaneous rate of change $\frac{dy}{dx}$ using the <u>delta method</u> for $y = 2x^2 - 3x + 5$

10. Using the delta method, find the slope of the curve $f(x) = x^2 - 3$, at $x = -2$

Derivatives: Find the <u>first derivative</u> for each function:

1. $y = 5x^3 + \sqrt{2}x - 3\pi$

2. $y = \ln(\cos(3x))$

3. $y = (5x - 3x^2)^{\frac{3}{2}}$

4. $V = e^{5t^2}$

5. $y = 2\ln(x^3 + x)$

6. $w = \cos(2\theta)\sin\theta$

7. $y = \sin^2(3x)$

8. $y = \frac{1}{2\sqrt{4-x^2}}$

9. $y = \frac{x^3}{2x+1}$

10. Find $\frac{dy}{dx}$ for $y^2 - 2x^2 + 2y = 5$

11. $y = x \sec(x^2)$

12. $y = \cot(x)\csc(3x)$

13. $y = 4\log_3 5x^2$

14. $y = \dfrac{2x-1}{(x-1)^2}$

15. Find the slope of the tangent to the curve $y = \dfrac{1}{x+1}$ at $x = 2$

16. Find the slope of the normal to this same curve at $x = 2$.

17. Find the slope of the tangent to the curve $2x^2 + 2y^3 - 9xy = 0$ at $(1, 2)$

18. If $y = (x^2 - x)^3$ find $y'(3)$

19. Find the differential of $y = -\dfrac{4}{x^2} + x - 7$

20. Write the equation, in general form, of the tangent and normal to each of the following curves:

a) $y = x^3 - 3x$ at $(2, 2)$ b) $x^2 + y^2 = 25$ at $(3, 4)$ c) $y = \ln (x + 1)^2$ at $x = 0$

21. Find the relative maxima/um, minima/um and point(s) of inflection, written as ordered pairs, for the function $y = x^3 + 3x^2 - 9x + 5$

22. Sketch the curve of the function above.

23. Is the curve $y = x^3 - 5x^2 - 7$ increasing or decreasing
    a) at $x = -1$ (justify your answer)
    b) at $x = 1$ (justify your answer)
    c) determine if the curve is concave up or concave down at $x = -2$ (justify your answer)

24. Indicate any horizontal or vertical asymptotes for the following:

a) $y = \dfrac{2x}{x-1}$ b) $y = \dfrac{1-3x}{2x+1}$

25. The height s, in feet, reached by a ball t seconds after being thrown vertically upward at 320 ft/s, is given by $s = 320t - 16t^2$. Find the greatest height reached by the ball, and the velocity with which it hits the ground.

26. A company finds that the cost C, in dollars, of producing x machine parts is given by $C = x^3 - 300x + 100$. Find the value of the instantaneous rate of change of C with respect to x when $x = 12$ parts.

27. A computer program increases the side of a square image on the screen at the rate of 0.25 in/sec. Find the rate at which the area of the image increases when the edge is 6.50 in.

28. The displacement s, in feet, of a body moving in a straight line, with initial velocity equal to zero, is given by $s = t^3 + 2t$ where t is time in sec.

a) Find the velocity at $t = 2$ sec. b) Find the acceleration at $t = 1$ sec.

29. A person walks toward the base of an 80-m high tower at the rate of 6 km/hr. At what rate is the person approaching the top of the tower when s/he is 40m from the base?

30. The area of a circular metal plate is increasing at the rate of 0.126 m^2/s. At what rate is the radius increasing when the radius is 2.00 m?

31. Find the dimensions of the largest rectangular box with square base and open top that can be made from 300 in^2 of metal.

Integration: Find the following indefinite integrals:

1. $y = \int -5x^{-3}dx$

2. $y = \int \left(9x^2 + x + 3\right)dx$

3. $v = \int \left(3t^2 - 2t\right)^{\frac{3}{2}}(6t - 2)dt$

4. $y = \int 2\sin(3\theta)\,d\theta$

5. $v = \int 2e^{2x^3}x^2dx$

6. $y = \int \frac{x\,dx}{1-2x^2}$

7. $y = \int \frac{6x^2dx}{\sqrt{2x^3+1}}$

8. $y = \int x\tan x^2dx$

9. $y = \int \frac{5}{(5x-3)^4}dx$

10. $y = \int \sqrt{4-x}\,dx$

11. Write the function y that has derivative $y' = 3x^2$ and passes through the point (-1, 7).

12. Write the function y that has derivative $y' = x + 1$ and passes through the point (-1, 4).

13. Given $y'' = 12$ find y' and y if $y' = 2$ when $x = 3$ and $y = 1$ when $x = 0$.

14. Evaluate the following definite integrals:

a) $\int_0^1 \frac{x\,dx}{4+x^2}$

b) $\int_0^4 \left(2x^2 + 4\right)dx$

15. Find the area under the curve $y = 4x^3 + 2x + 3$ between x = 0 and x = 1, and the x-axis.

16. Find the area bounded by $y = 2x + \frac{1}{x^2}$ and the x-axis from x = 1 to x = 4.

17. Find the area bound by the equations, $y = -x^2 + 4$, x = -1, x = 1 and y = 0.

ANSWERS TO FINAL REVIEW

Conic Sections:

conic section	general form	standard form	find
line	E) $x - 6y + 14 = 0$	I) $y = \frac{1}{6}x + \frac{7}{3}$	m = $\frac{1}{6}$ x int (-14, 0) y int $\left(0, \frac{7}{3}\right)$
circle	F) $x^2 + y^2 - 8y = 0$	K) $x^2 + (y - 4)^2 = 16$	C (0, 4) r = 4
parabola	C) $x^2 + 2x - y - 8 = 0$	G) $(x + 1)^2 = (y + 9)$	V (-1, -9) F $\left(-1, -8\frac{3}{4}\right)$ L = 1 dir y = $-9\frac{1}{4}$ AOS x = -1
parabola	D) $y^2 + 4x + 6y - 3 = 0$	L) $(y + 3)^2 = -4(x - 3)$	V (3, -3) F (2, -3) L = 4 dir x = 4 AOS y = -3
ellipse	B) $4x^2 + y^2 + 8x - 6y + 9 = 0$	H) $\frac{(x+1)^2}{1} + \frac{(y-3)^2}{4} = 1$	C (-1, 3)V (-1, 5) (-1, 1) F (-1, 4.7) (-1, 1.3) major axis vert = 4 minor axis = 2
hyperbola (B = 0)	A) $16x^2 - 9y^2 - 64x - 54y - 161 = 0$	J) $\frac{(x-2)^2}{9} - \frac{(y+3)^2}{16} = 1$	C (2, -3) V (5, -3) (-1, -3) F (7, -3) (-3, -3) transverse axis horiz = 6, conjugate axis = 8, slopes of asymptotes = $\pm\frac{4}{3}$

2. $(y - 4)^2 = 12(x + 1)$ 3. $(x-2)^2 = -\frac{4}{7}(y-5)$ 4. $\frac{(y+4)^2}{100} + \frac{(x-3)^2}{64} = 1$

5. $144x^2 - 25y^2 - 3600 = 0$ 6. $(x + 2)^2 + (y - 4)^2 = 136$

Limits:

1. 1 2. $+\infty$ 3. 2 4. 2x 5. $-\frac{1}{2}$ 6. 0

7. 0 8. $-\infty$

9. $y = 2x^2 - 3x + 5$

$$y' = \lim_{\Delta x \to 0} \frac{2(x + \Delta x)^2 - 3(x + \Delta x) + 5 - (2x^2 - 3x + 5)}{\Delta x}$$

$$y' = \lim_{\Delta x \to 0} \frac{2(x^2 + 2x(\Delta x) + (\Delta x)^2) - 3x - 3\Delta x + 5 - 2x^2 + 3x - 5}{\Delta x}$$

$$y' = \lim_{\Delta x \to 0} \frac{2x^2 + 4x(\Delta x) + 2(\Delta x)^2 - 3x - 3\Delta x + 5 - 2x^2 + 3x - 5}{\Delta x}$$

$$y' = \lim_{\Delta x \to 0} \frac{4x(\Delta x) + 2(\Delta x)^2 - 3\Delta x}{\Delta x}$$

$$y' = \lim_{\Delta x \to 0} 4x + 2(\Delta x) - 3 \Rightarrow y' = 4x - 3$$

10. slope = −4

Derivatives:

1. $y' = 15x^2 + \sqrt{2}$

2. $y' = \dfrac{-3\sin 3x}{\cos 3x} = -3\tan(3x)$

3. $y' = \dfrac{3}{2}(5-6x)(5x-3x^2)^{1/2}$

4. $y' = 10te^{5t^2}$

5. $y' = \dfrac{2(3x^2+1)}{x^3+x}$

6. $w' = \cos(2\theta)\cos\theta - 2\sin\theta\sin(2\theta)$

7. $y' = 6\sin(3x)\cos(3x)$

8. $y' = \dfrac{x}{2(4-x^2)^{3/2}}$

9. $y' = \dfrac{4x^3+3x^2}{(2x+1)^2}$

10. $\dfrac{dy}{dx} = \dfrac{2x}{y+1}$

11. $\sec(x^2) + 2x^2\sec(x^2)\tan(x^2)$

12. $-\csc^2(x)\csc(3x) - 3\csc(3x)\cot(3x)\cot(x)$

13. $y' = \dfrac{8x}{x^2 \ln 3}$

14. $y' = \dfrac{-2x}{(x-1)^3}$

15. $m = -\dfrac{1}{9}$

16. m = 9

17. $y' = -\dfrac{22}{15}$

18. y'(3) = 540

19. $dy = \left(\dfrac{8}{x^3}+1\right)dx$

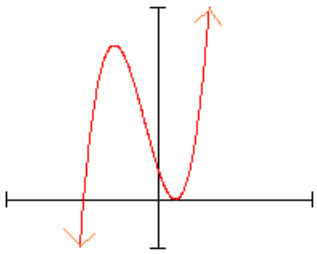
20. a) tan: 9x – y – 16 = 0; normal: x + 9y – 20 = 0

    b) tan: 3x + 4y – 25 = 0; normal: 4x – 3y = 0

    c) tan: 2x – y = 0; normal: x + 2y = 0

21. Max (–3, 32); Min (1,0) POI (–1, 16)

22.



23. a) increasing, y' > 0 b) decreasing y' < 0 c) concave down y'' < 0

24. a) HA: y = 2 VA: x = 1 b) HA: $y = -\dfrac{3}{2}$ VA: $x = -\dfrac{1}{2}$

25. Greatest height = 1600 ft Velocity when hits the ground = -320 ft/sec

26. C'(12) = \$132 / part

27. $\dfrac{dA}{dt} = 3.25\text{in}^2/\text{sec}$

28. v(2) = 14 ft/sec a(1) = 6 ft/sec^2

29. Approaching top of the tower at a rate of ≈ 2.7 km/hr

30. 0.010 m/s

31. 10-in x 10-in x 5-in

Integration:

1. $y = \frac{5}{2}x^{-2} + C$
2. $y = 3x^3 + \frac{x^2}{2} + 3x + C$
3. $v = \frac{2}{5}(3t^2 - 2t)^{5/2} + C$
4. $y = -\frac{2}{3}\cos(3\theta) + C$
5. $v = \frac{1}{3}e^{2x^3} + C$
6. $y = -\frac{1}{4}\ln\left|1 - 2x^2\right| + C$
7. $y = 2\sqrt{2x^3 + 1} + C$
8. $y = -\frac{1}{2}\ln\left|\cos x^2\right| + C$
9. $y = -\frac{1}{3(5x - 3)^3} + C$
10. $y = -\frac{2}{3}(4 - x)^{3/2} + C$
11. $y = x^3 + 8$
12. $x^2 + 2x - 2y + 9 = 0$
13. $y = 6x^2 - 34x + 1$
14. a) 0.112 b) $\frac{176}{3}$
15. A = 5 sq units
16. A = 15.75 sq units
17. A = $\frac{22}{3}$ sq units