MATH 150
Final Exam Review

1. Simplify:

a) $\dfrac{(-3)+(-2)(-10)}{-(2)(6)-5}$ b) $4y - 2[3x - 2(x - y)]$ c) $12x - 5[3y - 2(y - x) + 12x]$

2. Evaluate:

a) $\left(3x^{a}\right)^{2a}$ when a = 2 and x = 3. Write as a power of 3.

b) $\left(3x^{2}\right)^{a}$ when a = –2 and x = 2 c) $\left(5x^{-3}\right)^{-a}$ when x = 2 and a = –2

3. Factor completely:

a) $16x^4 - 1$ b) $2x^2 - 3x - 2$ c) $3x^5 - 48x$

d) $125x^6 - 27y^9$ e) $16x^3 + 2y^{12}$

4. Reduce to lowest terms:

a) $\dfrac{16 - a^2}{a^2 - 8a + 16}$ b) $\dfrac{7x^2 - 14x}{14 - 5x - x^2}$ c) $\dfrac{14x^2y^5 + 7xy^3}{7xy}$

5. Simplify:

a) $\dfrac{\frac{1}{x} - \frac{1}{y}}{\frac{1}{x} + \frac{1}{y}}$ b) $\dfrac{x + \frac{1}{x}}{1 - \frac{1}{x}}$ c) $\dfrac{\frac{x}{yz}}{\frac{x}{y} + \frac{y}{z}}$

6. Rationalize the denominator and simplify:

a) $\dfrac{1}{2}\sqrt{\dfrac{3}{8x}}$ b) $\dfrac{2}{3}\sqrt{\dfrac{1}{2}}$ c) $5\sqrt{\dfrac{3}{5}}$

d) $\dfrac{5}{\sqrt{5} - 3}$ e) $\dfrac{3x + 2}{\sqrt{2} - 2}$

7. a) Evaluate $\left(2x - \sqrt{y}\right)^2$ when x = 2 and y = 3 b) Simplify: $\left(5 + \sqrt{13}\right)\left(5 - \sqrt{13}\right)$

For c) and d) simplify and put your answers in a + bi form. c) $(3 - 4i)^2$ d) $\dfrac{7}{5 + 2i}$

8. Solve for x: a) $a = \dfrac{b + x}{abx}$ b) $c = \dfrac{1}{2}\left[f - 2ax\right]$

9. Given: (1) $f(x) = \frac{3x^2}{2} + 4$ find: a) f(0) b) f(–2) c) f(3)

(2) $f(x) = (2x + 1)^2 - 5$ find: a) f(–1) b) f(0)

10. Find the slope of the line passing through the points (–8, 3) and (–4, –6).

11. Find the slope of the line with equation $3x + 2y - 9 = 0$

12. Find the y intercept, as an ordered pair, of the line whose equation is $12x + 5y = 60$.

13. Find the slope of a line parallel to the line with equation $3x - y = 12$.

14. Find the slope of a line perpendicular to the line with equation $4x - 2y = 13$.

15. Find the slope of a line having an angle of inclination of 42°. Round to four decimal places.

16. Find the angle of inclination, to the nearest degree, of a line whose slope is 0.8666.

17. Find the equation of a line with a slope of $\frac{2}{3}$ and passing through the point (–9, 8). Give your answer in general and slope–intercept form.

18. Find the equation of a line that is parallel to the line with equation $3x - y - 12 = 0$ and whose x intercept is (7, 0). Give your answer in general and slope–intercept form.

19. Find the equation of a line passing through (–3, 5) and (2, –7). Give your answer in general form.

20. Find the equation of a line passing through (–4,2) and perpendicular to $10x - 2y - 6 = 0$. Give your answer in general form.

21. Find the equation of a line passing through (–3, 8) and parallel to the y–axis. (This does not have to be in general form.)

22. Find the distance between the points (–5,4) and (6,–2). Give your answer in simplified radical form.

23. Solve the following system of equations for y: $5x - 3y = -25$ and $x + 2y = -3$.

24. Find the arc length intercepted by a central angle of 52° in a circle of radius 7.0 inches. Round to the nearest tenth.

25. Simplify and express the answer with positive exponents:

a) $(-5x^2y^{-3})^2$ b) $(-6x^{-3}y^4)(5xy^{-2})$ c) $\frac{-42x^5y^{-6}}{7x^{-2}y}$

26. If $\log_3 x = 5$, find x.

27. If $\log_5 125 = y$, find y.

28. If $\log_b 125 = \frac{3}{2}$, find b.

29. If $\log_b 2 = \frac{1}{5}$, find b.

30. Rewrite $3 \log x - \log 5$ as a single positive logarithm whose coefficient is one.

For 31–33, either state the restrictions or check. All answers need to be in simplest form.

31. Solve for x: $3 \log x - 2 \log (2x) = 2 \log 5$

32. Solve for x, exactly: $3 \log (x^2 + 2) - 6 = 0$

33. Solve for x: $\log (5x + 2) - 1 = \log (2x - 1)$

34. Given: $\log x^3 = .609$ find: a) $\log x^2$ b) $\log \sqrt{x}$ Give your answers to three decimal places.

35. Evaluate exactly: $\ln e^{1.4}$

36. The graph of $y = 7x^2 - 2x - 5$ intersects the x axis at what point(s)?

37. Solve for w by completing the square: $w^2 - 6w - 3 = 0$.

38. Solve for y: $y^2 - 3y + 1 = 0$. Use the quadratic formula and give your answer in simplified radical form.

39. Convert to radians: Round to three decimal places.

a) 340° b) 150° c) 320° d) 270°

40. Convert to degrees:

a) $\frac{4\pi}{9}$ rad b) $\frac{8\pi}{9}$ rad c) 5π radians d) 1.27 radians

41. Find the acute angle, to the nearest degree, whose secant is 4.5369

42. Find the acute angle, to the nearest degree, whose cos is 0.9812

43. Find the value of: a) sin 275.2° b) cos 200.5° c) sec 118.4° d) csc 315.5°

44. Write $\sin 40^\circ$ as an equivalent co–function.

45. Find cot 50°. Round to four decimal places.

46. For a) – d) round to the nearest degree.

a) Find the first quadrant angle whose csc is 1.0349

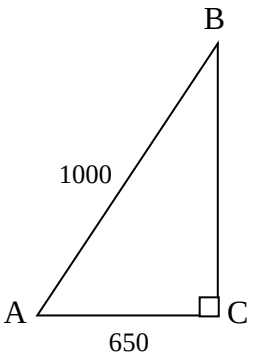
b) Find the second quadrant angle whose tan is –2.1445

c) Find the third quadrant angle whose sin is –0.6428

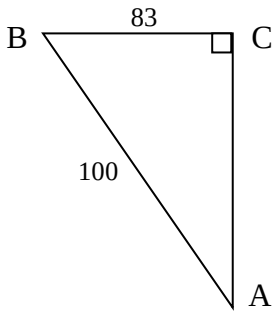
d) Find the fourth quadrant angle whose cos is .5

47. In right ΔABC, find angle A, to the nearest tenth of a degree.

a)



b)



48. If sin x = 0.4848, find csc x. Round to four decimal places.

49. If tan x = 0.6008, find cot x. Round to four decimal places.

50. Given oblique triangle ABC with c = 21, a = 14 and C = 104.5°. Find angle A, rounded to the nearest tenth of a degree.

51. Given oblique triangle ABC, with side a = 12.5, side b = 9.2 and C = 50°. Find side c, rounded to the nearest tenth.

52. Given oblique triangle ABC with a = 20, C = 110^o and A = 42°. Find side c, rounded to the nearest whole number.

53. If $\sin\theta = -\frac{3}{5}$, and θ is in quadrant IV, find $\cot\theta$ as a simple fraction.

54. For the parabola, $y = -x^2 + 4x - 3$:

a) find the equation of axis of symmetry
b) find the coordinates of turning point (vertex)
c) find the x–intercept(s) (zeroes) as an ordered pair(s)
d) find the y intercept as an ordered pair
e) sketch the graph

55. Calculate the discriminant and determine the nature of the roots of the equation $7x = -3x^2 + 6$

56. Given the equation of a line $5x - 4y - 20 = 0$:

a) find the slope
b) find the x intercept as an ordered pair
c) find the y intercept as an ordered pair
d) sketch the line

57. Simplify:

a) $\sqrt{4a} + \sqrt{16a} - \sqrt{25a}$

b) $\sqrt{27} - 3\sqrt{12} + 5\sqrt{48}$

c) $\left(\sqrt{3} + 2\right)\left(\sqrt{5} - 4\right)$

d) $\sqrt[3]{32x^2y^4}$

58. Solve for x and check:

a) $2\sqrt{x - 5} = 20$

b) $\sqrt{3x + 1} - 5 = 3$

59. Find the domain of the following:

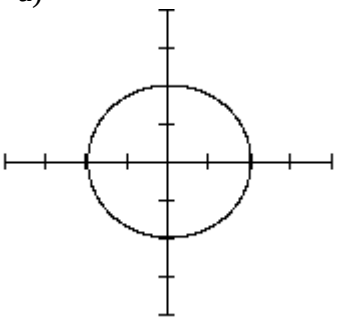
a) $y = x^2 + 2$

b) $y = \sqrt{3x - 4}$

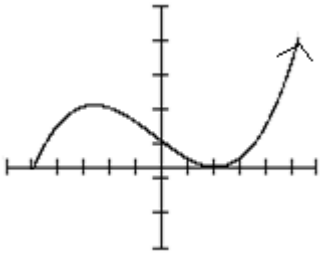
c) $y = \dfrac{x}{x^2 + x - 6}$

60. Find the domain and range of the following. Each mark represents one unit.

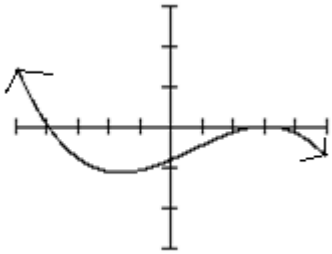
a)



b)



c)



61. If $\ln(ex) = 4$, find: a) $\ln x$ b) $\ln x^3$ c) $\ln \sqrt[3]{x}$

62. Solve for x:

a) $\log_x 2 = -\dfrac{1}{3}$

b) $25^{2-x} = \left(\dfrac{1}{5}\right)^x$

c) $\log_{27} x = -\dfrac{4}{3}$

d) $\log_x\left(\dfrac{1}{9}\right) = -\dfrac{2}{3}$

e) $8^x = 15$

f) $27 = 9^{x-5}$

g) $\log(x + 5) + \log 2 = 2\log(x + 5)$

h) $\log(x + 5) + \log 2 = 2$

63. Simplify:

a) $\frac{3}{x}-\frac{5}{x+2}+\frac{x}{2}$ b) $\frac{x^2+5x+6}{6x-30}\cdot\frac{3x-6}{x+3}\div\frac{x^2-4}{5x-x^2}$ c) $\frac{6}{2p-4}-\frac{4}{p-2}+\frac{1}{p^2-4}$

64. Factor completely:

a) $x^2 - 13xy + 36y^2$ b) $a^8 - b^8$ c) $42x^2 - 24x - 18$ d) $ab - ay - bx + xy$

65. Solve these literal equations for x: a) $p(x - b) = qx + d$ b) $\frac{bx-c}{ax-c}=5$

66. a) If $f(x) = -x^2 + 5x - 3$, find $f(-2)$ b) If $g(x) = x^3 + 1$ and $h(x) = 2x + 5$, find $\frac{g(2)+3h(1)}{h(-4)}$

67. If $f(x) = x^2 + 5$ and $g(x) = 3x - 7$, find, in simplest form: a) $f(g(x))$ b) $g(f(x))$

68. Divide using long division and write any remainder as a fraction: $(x^3 - 2x - 25) \div (x - 3)$

69. Solve for x: $\frac{2}{x-3}+\frac{x}{x^2-9}=\frac{4}{x+3}$. Either state the restrictions or show the check.

70. Solve the following system for x, y, and z.

$$\begin{aligned} x + 2y + z &= 3 \\ 2x - y + 2z &= 6 \\ 3x + y - z &= 5 \end{aligned}$$

71. A lamp pole casts a shadow 53.0 feet long when the angle of elevation of the sun is 48.1°. Find the height of the lamp pole, rounded to the nearest tenth.

72. One pipe can fill a certain tank in 6 hours working alone and another in 4 hours. If both pipes will be running at the same time, how long will it take the pipes to fill the tank, working together?

73. A train departs at noon travelling at a speed of 64 km/hr. A car leaves the same station half an hour later to overtake the train, travelling on a road parallel to the track. If the car's speed is 96 km/hr, at what distance from the station will it overtake the train?

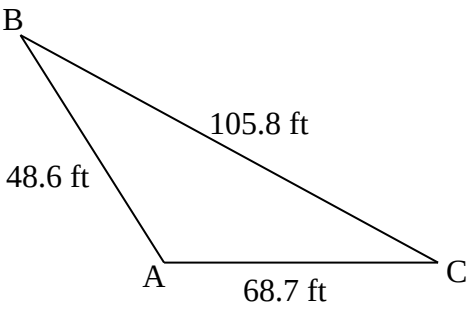
74. A certain sum of money is split among three people. The first gets one–third of the money, the second gets one–fourth of what is left, and then the third gets the rest, which is $360. How much money was there to start?

75. Solve the following system *graphically* for x and y: $6x - 2y - 2 = 0$ and $x + y - 3 = 0$

76. Sketch and state the amplitude and period: a) $y = 3 \cos x$ b) $y = -2 \sin x$

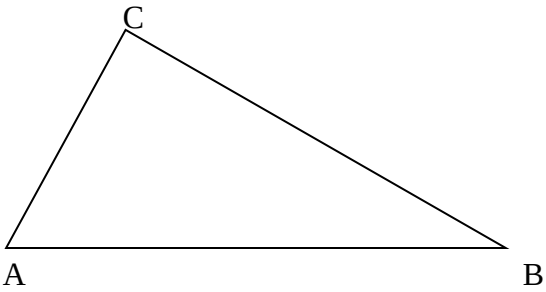
77. Find the inverse of: a) $f(x) = 7 - 5x$ b) $y = 8x^3 + 3$

78.



A survey team measured the distances between three benchmarks but neglected to measure the angles. Find the size of angle A, rounded to the nearest tenth of a degree, using the measurements taken by the survey team.

79.



An electric line 243.5 meters long is connected between points A and B and one 142.9 meters long is between points A and C. If angle B measures 32.4 degrees, find angle C, to the nearest tenth of a degree. Note: the triangle is not drawn to scale.

Use the following formulas for 80 and 81. Exponential Growth: $y = ae^{nt}$ and Exponential Decay: $y = ae^{-nt}$

80. How many years, rounded to the nearest tenth, will it take for an initial deposit of $1500 to yield $2500 if it is invested at an interest rate of 2.5% per year, compound continuously?

81. A population is decreasing at a rate of 3.6% per year from an initial population of 9000. How many years, rounded to the nearest tenth of a year, will it take the population to decrease to one–third of the initial amount?

For questions 82 and 83, tell if the function has a maximum or minimum value, how you can tell from the equation, and the point (x, y) at which this occurs.

82. $y = x^2 + 8x + 10$

83. $y = -4x^2 - 16x + 23$

84. A ball is tossed upward with an initial velocity of 64 ft/s from a height of 4 feet. The height of the ball, h, after t seconds can be modeled by the quadratic function $h = -16t^2 + 64t + 4$.

a) At what time is the maximum height reached? What is the maximum height?
b) How long does the ball remain in the air? Round to the nearest hundredth of a second.
c) In how many seconds, rounded to the nearest tenth, will the ball be at a height of 50 feet? Why are there two possible solutions?

Answers to Final Exam Review

1) a) -1 b) $-2x$ c) $-58x - 5y$

2) a) 3^{12} b) $\frac{1}{144}$ c) $\frac{25}{64}$

3) a) $(2x - 1)(2x + 1)(4x^2 + 1)$ b) $(2x + 1)(x - 2)$ c) $3x(x - 2)(x + 2)(x^2 + 4)$
d) $(5x^2 - 3y^3)(25x^4 + 15x^2y^3 + 9y^6)$ e) $2(2x + y^4)(4x^2 - 2xy^4 + y^8)$

4) a) $-\frac{a+4}{a-4}$ b) $-\frac{7x}{x+7}$ c) $2xy^4 + y^2$

5) a) $\frac{y-x}{x+y}$ b) $\frac{x^2+1}{x-1}$ c) $\frac{x}{xz+y^2}$

6) a) $\frac{\sqrt{6x}}{8x}$ b) $\frac{\sqrt{2}}{3}$ c) $\sqrt{15}$ d) $\frac{5\sqrt{5}+15}{-4}$ e) $\frac{6x+4+3x\sqrt{2}+2\sqrt{2}}{-2}$

7) a) $19 - 8\sqrt{3}$ b) 12 c) $-7 - 24i$ d) $\frac{35}{29} - \frac{14}{29}i$

8) a) $x = \frac{b}{a^2b - 1}$ b) $x = \frac{f - 2c}{2a}$

9) (1) a) 4 b) 10 c) $\frac{35}{2}$
(2) a) -4 b) -4

10) $m = -\frac{9}{4}$ 11) $m = -\frac{3}{2}$ 12) $(0, 12)$ 13) $m = 3$

14) $m = -\frac{1}{2}$ 15) 0 .9004 16) $41°$ 17) $2x - 3y + 42 = 0$; $y = \frac{2}{3}x + 14$

18) $3x - y - 21 = 0$; $y = 3x - 21$ 19) $12x + 5y + 11 = 0$ 20) $x + 5y - 6 = 0$

21) $x = -3$ 22) $\sqrt{157}$ 23) $y = \frac{10}{13}$ 24) 6.4 in.

25) a) $\frac{25x^4}{y^6}$ b) $\frac{-30y^2}{x^2}$ c) $\frac{-6x^7}{y^7}$

26) $x = 243$ 27) $y = 3$ 28) $b = 25$ 29) $b = 32$

30) $\log\left(\frac{x^3}{5}\right)$ 31) $x = 100$ 32) $x = \pm 7\sqrt{2}$ 33) $x = \frac{4}{5}$

34) a) 0.406 b) 0.102 35) 1.4 36) $\left(-\frac{5}{7}, 0\right)$ and (1, 0)

37) $w = 3 \pm 2\sqrt{3}$ 38) $x = \frac{3 \pm \sqrt{5}}{2}$

39) a) $\frac{17\pi}{9} \approx 5.934$ b) $\frac{5\pi}{6} \approx 2.618$ c) $\frac{16\pi}{9} \approx 5.585$ d) $\frac{3\pi}{2} \approx 4.712$

40) a) 80° b) 160° c) 900° d) 72.8°

41) 77° 42) 11°

43) a) –0.9959 b) –0.9367 c) –2.1025 d) –1.4267

44) $\cos 50^o$ 45) 0.8391

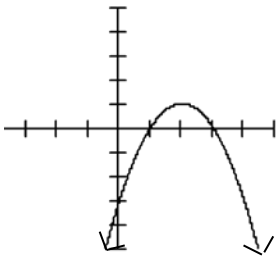
46) a) 75° b) 115° c) 220° d) 300°

47) a) 49.5° b) 56.1° 48) 2.0627 49) 1.6644

50) c = 40.2° (reject 139.8°) 51) c = 9.6 52) c = 28

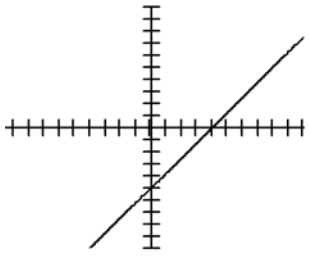
53) $-\frac{4}{3}$

54) a) x = 2 b) (2, 1) c) (3, 0) (1, 0) d) (0, –3) e) Each mark represents one unit.



55) $b^2 - 4ac = 121$; roots are real, rational and unequal

56) a) $m = \frac{5}{4}$ b) (4, 0) c) (0, –5) d) Each mark represents one unit.



57) a) $\sqrt{a}$ b) $17\sqrt{3}$ c) $\sqrt{15}-4\sqrt{3}+2\sqrt{5}-8$ d) $2y\sqrt[3]{4x^2y}$

58) a) x = 105 b) x = 21

59) a) all reals b) $x \geq \frac{4}{3}$ c) all reals except 2 and –3 or $x \neq 2$ and $x \neq -3$

60) a) Domain: $-2 \leq x \leq 2$
Range: $-2 \leq y \leq 2$
b) Domain: $x \geq -5$
Range: $y \geq 0$
c) Domain: all reals
Range: all reals

61) a) 3 b) 9 c) 1

62) a) $x = \frac{1}{8}$ b) x = 4 c) $x = \frac{1}{81}$ d) x = 27
e) x = 1.302 f) $x = \frac{13}{2}$ g) x = –3 h) x = 45

63) a) $\frac{x^3+2x^2-4x+12}{2x(x+2)}$ b) $-\frac{x}{2}$ c) $\frac{-p-1}{(p+2)(p-2)}$

64) a) (x – 9y)(x – 4y) b) $(a-b)(a+b)(a^2+b^2)(a^4+b^4)$
c) 6(7x + 3)(x – 1) d) (b – y)(a – x)

65) a) $x = \frac{pb+d}{p-q}$ b) $x = \frac{4c}{5a-b}$

66) a) –17 b) –10

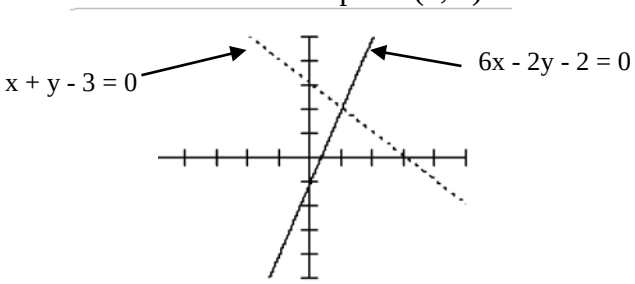
67) a) $9x^2 - 42x + 54$ b) $3x^2 + 8$

68) $x^2 + 3x + 7 - \frac{4}{x-3}$

69) x = 18 70) (2, 0, 1) 71) 59.1 feet

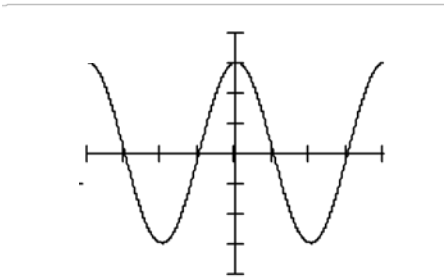
72) $\frac{12}{5}$ or 2.4 hours 73) 96 km. 74) $720

75) The two lines intersect at the point (1, 2) Each mark represents one unit.

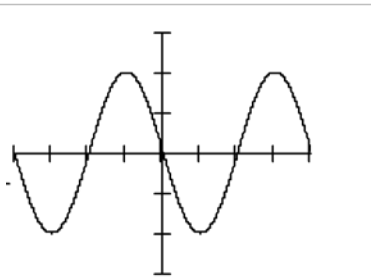


76) The curves are graphed from -2π to 2π (-360^o to 360^o) Each mark on the y axis represents one unit.

a) $y = 3 \cos x$
amplitude is 3; period is 2π or 360^o



b) $y = -2 \sin x$
amplitude is 2; period is 2π or 360^o



77) a) $f^{-1}(x) = \dfrac{7 - x}{5}$ b) $f^{-1}(x) = \dfrac{\sqrt[3]{x - 3}}{2}$

78) 128.0 degrees

79) 65.9 or 114.1 degrees

80) 20.4 years

81) 30.5 years

82) min; the leading coefficient is positive; (–4, –6)

83) max; the leading coefficient is negative; (–2, 39)

84) a) 2 seconds; 68 feet
b) 4.06 seconds
c) 3.1 and 0.9 seconds; reaches 50 on the way up and on the way down