Math 170 Problem of the Week for Week 5

Solve for x, $\left(1+\sqrt{1+x}\right)^3 = x^2$

$$\left(\sqrt{1+\sqrt{1+x}}\right)^6 = \left(\sqrt[3]{x}\right)^6$$

(I will put a hint on a separate link. This will be a link to a strategy)

The method I used, towards the end, involves the rational zero test and synthetic/long division

let $u = \sqrt{1+x}$ — get in terms of u with two identities circled.

$u^2 = 1+x$

$u^2 - 1 = x$

$(u^2-1)^2 = x^2$

$(1+u)^3 = (u^2-1)^2$ expand

$1 + 3u + 3u^2 + u^3 = u^4 - 2u^2 + 1$ set $= 0$

$u^4 - u^3 - 5u^2 - 3u = 0$

$u(u^3 - u^2 - 5u - 3) = 0$

$u = 0$

$\therefore$

$0 = \sqrt{1+x}$

$x = -1$

Doesn't check

$$\begin{array}{r|rrrr} 3 & 1 & -1 & -5 & -3 \\ & \downarrow & 3 & 6 & 3 \\ \hline & 1 & 2 & 1 & 0! \end{array}$$

$u = 3$

$3 = \sqrt{1+x}$

$9 = 1+x$

$x = 8$

checks!

$u^2 + 2u + 1 = 0$

$(u+1)(u+1) = 0$

$u = -1$

$-1 = \sqrt{1+x}$

$x = 0$

doesn't check