

Math 150, Problem of the week for Week 5

Hint: The answers are irrational, but real.

Solve for x.

$$\frac{3^{x^2}}{9^x} = 81$$

① turn to powers of three

$$\frac{3^{x^2}}{3^{2x}} = 3^4 \Rightarrow 3^{x^2-2x} = 3^4$$

② combine left side

③ Set exponents equal, via one-to-one property

$$x^2 - 2x = 4 \quad (\text{or complete the square}) \rightarrow x^2 - 2x + 1 = 4 + 1$$

-4 -4

$$x^2 - 2x - 4 = 0$$

$$x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(-4)}}{2(1)}$$

$$x = \frac{2 \pm \sqrt{20}}{2}$$

$$x = \frac{2 \pm 2\sqrt{5}}{2} \Rightarrow 1 \pm \sqrt{5}$$

$$\sqrt{(x-1)^2} = \sqrt{5}$$

$$(x-1) = \pm\sqrt{5}$$

$$x = 1 \pm \sqrt{5}$$